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Anti-Periodic Boundary Value Problem for Third Order Nonlinear Impulsive q_k-Integrodifference Equation

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Abstract:

A third order nonlinear impulsive integrodifference equation within the frame of q_k - quantum calculus is investigated by applying using fixed point theorems. The conditions for existence and uniqueness of solution are obtained.

Keywords: q_k - Integrodifference Equation, q_k - derivatives, q_k - integrals, Boundary Value Problem.

INTRODUCTION

The $q-$calculus was initiated in twenties of the last century. However, it has gained considerable popularity and importance during the last three decades or so. Mathematical modeling of a wide variety of practical situations and has emerged as an important area of investigation in recent years. Their study has not only important theoretical meaning but also wide applications in conformal quantum mechanics, high energy physics, etc. We refer the reader to recent articles [1-7]. Recently, in [8], authors research first order nonlocal boundary value problem for nonlinear impulsive q_k- integrodifference equation, respectively. In this paper, we study the existence and uniqueness of solutions for third order nonlinear q_k-integrodifference equation with anti-periodic boundary condition and impulses:

$$\begin{cases} D_{q_k}^3 u(t) = f(t,u(t)), & 0<q_k<1, t\in J' \\ \Delta u(t_k) = I_k(u(t_k)), & k=1,2,\cdots,p \\ D_{q_k} u(t_k^{+}) - D_{q_{k-1}} u(t_k^{-}) = J_k(u(t_k)), & k=1,2,\cdots,p \\ D_{q_k}^2 u(t_k^{+}) - D_{q_{k-1}}^2 u(t_k^{-}) = L_k(u(t_k)), & k=1,2,\cdots,p \\ u(0)=-u(1),\ \ D_{q_0}u(0) = -D_{q_{p+1}}u(1),\ \ D_{q_0}^2 u(0) = -D_{q_{p+1}}^2 u(1) & \end{cases} \tag{1}$$

where D_{q_k} is q_k-derivatives $(k=0,1,\cdots,p+1)$, respectively. $f\in C(J\times\mathbb{R},\mathbb{R}), I_k, J_k, L_k \in C(\mathbb{R},\mathbb{R})$

$J=[0,1]$, $0=t_0<t_1<t_2<\cdots<t_p<t_{p+1}=1$, $J'=[0,1]\setminus\{t_1,t_2,\cdots,t_p\}$, $\Delta u(t_k)=u(t_k^+)-u(t_k^-)$ .Where $u(t_k^+)$ and $u(t_k^-)$ denote the right and the left limits of $u(t)$ at $t=t_k (t=1,2,\cdots,p)$, respectively.

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PRELIMINARIES

Let $J_0=[0,t_1], J_1=(t_1,t_2],\cdots,J_{p-1}=(t_{p-1},t_p], J_p=(t_p,1]$ and introduce the space:

$PC(J,\mathbb{R})=\{u:J\to\mathbb{R}\ u\in C(J_k)$,and $u(t_k^+)$ and $u(t_k^-)$ exist with $u(t_k^-)=u(t_k)$, $k=1,2,\cdots,p\}$.

$PC^1(J,\mathbb{R})=\{u:J\to\mathbb{R}\ u\in C(J_k), D_{qk}u(t_k^-), D_{qk}\,u(t_k^+)$ exist and $D_{q_k}x(t)$ is left continuous at t_k ,for $k=1,2,\cdots,p\}$ .And $PC^2(J,\mathbb{R})=\{u:J\to\mathbb{R}\ u\in C^2(J_k), k=1,2\cdots\cdots,p, u(t_k^+)$, $D_{q_k}u(t_k{}^+)$, $D_{q_k}^2u(t_k{}^+)$, $D_{q_k}^2u(t_k{}^-)$ exist and $D_{q_k}^2u(t_k)$ is left continuous at t_k ,for $k=1,2,\cdots,p\}$ with the norm $\|u\|_{PC^2}=\sup_{t\in J}\left\{\|u\|_{PC},\|D_{q_k}u\|_{PC},\|D_{q_k}^2u\|_{PC}\right\}$.Then $PC(J,\mathbb{R}), PC^1(J,\mathbb{R}), PC^2(J,\mathbb{R})$, are Banach space.

Definition A function $u\in PC^2(J,\mathbb{R})$with its derivative of third order existing on J is a solution of (1) if it satisfies (1).

For convenience, let us recall some basic concepts of q_k-calculus [9].

For $0<q_k<1$, $t\in J_k$,we define the q_k-derivatives of a real valued continuous function f as

$$D_{q_k}f(t)=\frac{f(t)-f(q_kt+(1-q_k)t_k)}{(1-q_k)(t-t_k)}\quad,\quad D_{q_k}f(t_k)=\lim_{t\to t_k}D_{q_k}f(t).$$

Higher order q_k-derivatives are given by $D_{q_K}^0f(t)=f(t)$, $D_{q_K}^nf(t)=D_{q_k}D_{q_k}^{n-1}f(t)$, $n\in N, t\in J_k$.

The q_k-integral of a function f is defined by

$${}_{t_k}I_{q_k}f(t):=\int_{t_k}^t f(s)d_{q_k}s=(1-q_k)(t-t_k)\times\sum_{n=0}^{\infty}q_k^n f(q_k^nt+(1-q_k)t_k)\quad,\ t\in J_k.$$

Provided the series converges. If $a\in(t_k,t)$ and f is defined on the interval (t_k,t), then

$$\int_a^t f(s)d_{q_k}s=\int_{t_k}^t f(s)d_{q_k}s-\int_{t_k}^a f(s)d_{q_k}s,\quad D_{q_k}\left({}_{t_k}I_{q_k}f(t)\right)=D_{q_k}\int_{t_k}^t f(s)d_{q_k}s=f(t),$$

$${}_{t_k}I_{q_k}\left(D_{q_k}f(t)\right)=\int_{t_k}^t D_{q_k}f(s)d_{q_k}s=f(t),\ {}_aI_{q_k}\left(D_{q_k}f(t)\right)=\int_a^t D_{q_k}f(s)d_{q_k}s=f(t)-f(a),\quad a\in(t_k,t).$$

For convenience, we arrange $\sum_{i=j}^{k}a_i=0$, for $k<j$.

Lemma 1. The function $u\in PC^2(J,\mathbb{R})$ is a solution of the impulsive q_k- integrodifference equation (1) if and only if u satisfies the q_k-integral equation

$$
\begin{aligned}
&u(t)= \\
&-\frac{1}{2}\Bigg[\sum_{i=1}^{p} I_i(u(t_i))+\sum_{i=1}^{p}(t-t_i)J_i(u(t_i))+\sum_{i=1}^{p}(t-t_i)^2 L_i(u(t_i))-\sum_{i=1}^{p-1}(t-t_P)(t_p-t_i)L_i(u(t_i)) \\
&-\sum_{i=1}^{p-2}\sum_{j=i+1}^{p-1}(t_{j+1}-t_j)(t_j-t_i)L_i(u(t_i))+\sum_{i=0}^{p-1}(t-t_{i+1})^2\int_{t_i}^{t_{i+1}} f(s,u(s))dq_i s-\sum_{i=0}^{p-2}(t-t_P)(t_p-t_{i+1})\int_{t_i}^{t_{i+1}} f(s,u(s))dq_i s \\
&-\sum_{i=0}^{p-3}\sum_{j=i+2}^{p-1}(t_{j+1}-t_j)(t_j-t_{i+1})\int_{t_i}^{t_{i+1}} f(s,u(s))dq_i s+\sum_{i=0}^{p-1}(t-t_{i+1})\int_{t_i}^{t_{i+1}}[(t_{i+1}-t_i)-q_i(s-t_i)]f(s,u(s))dq_i s \\
&+\sum_{i=0}^{p-1}\int_{t_i}^{t_{i+1}}[(t_{i+1}-t_i)-q_i(s-t_i)]^2 f(s,u(s))dq_i s+\int_{t_p}^{1}[(1-t_p)-q_p(s-t_p)]^2 f(s,u(s))dq_p s\Bigg] \\
&+\sum_{i=1}^{k} I_i(u(t_i))+\sum_{i=1}^{k}(t-t_i)J_i(u(t_i))+\sum_{i=1}^{k}(t-t_i)^2 L_i(u(t_i))-\sum_{i=1}^{k-1}(t-t_k)(t_k-t_i)L_i(u(t_i)) \\
&-\sum_{i=1}^{k-2}\sum_{j=i+1}^{k-1}(t_{j+1}-t_j)(t_j-t_i)L_i(u(t_i))+\sum_{i=0}^{k-1}(t-t_{i+1})^2\int_{t_i}^{t_{i+1}} f(s,u(s))dq_i s-\sum_{i=0}^{k-2}(t-t_k)(t_k-t_{i+1})\int_{t_i}^{t_{i+1}} f(s,u(s))dq_i s \\
&-\sum_{i=0}^{k-3}\sum_{j=i+2}^{k-1}(t_{j+1}-t_j)(t_j-t_{i+1})\int_{t_i}^{t_{i+1}} f(s,u(s))dq_i s+\sum_{i=0}^{k-1}(t-t_{i+1})\int_{t_i}^{t_{i+1}}[(t_{i+1}-t_i)-q_i(s-t_i)]f(s,u(s))dq_i s \\
&+\sum_{i=0}^{k-1}\int_{t_i}^{t_{i+1}}[(t_{i+1}-t_i)-q_i(s-t_i)]^2 f(s,u(s))dq_i s+\int_{t_k}^{t}[(t-t_k)-q_k(s-t_k)]^2 f(s,u(s))dq_k s \\
&:=\Gamma[u(t), f(t,u(t))]. \qquad (2)
\end{aligned}
$$

Proof. Let u be a solution of q_k – difference equation (1). For $t\in J_0$, applying the operator ${}_0I_{q_0}$ on both sides of $D^3_{q_k}u(t)=f(t,u(t))$, we have

$$
\begin{aligned}
D^2_{q_0}u(t)&=D^2_{q_0}u(0^+)+{}_0I_{q_0}y_{q_0}(t)=D^2_{q_0}u(0^+)+\int_0^t f(s,u(s))d_{q_0}s \\
D_{q_0}u(t)&=D_{q_0}u(0^+)+tD^2_{q_0}u(0^+)+\int_0^t[t-q_0 s]f(s,u(s))d_{q_0}s \\
u(t)&=u(0^+)+tD_{q_0}u(0^+)+t^2D^2_{q_0}u(0^+)+\int_0^t[t-q_0 s]^2 f(s,u(s))d_{q_0}s.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
D^2_{q_0}u(t_1^-)&=D^2_{q_0}u(0^+)+\int_0^{t_1} f(s,u(s))d_{q_0}s \\
D_{q_0}u(t_1^-)&=D_{q_0}u(0^+)+t_1D^2_{q_0}u(0^+)+\int_0^{t_1}[t_1-q_0 s]f(s,u(s))d_{q_0}s \\
u(t_1^-)&=u(0^+)+t_1D_{q_0}u(0^+)+{t_1}^2D^2_{q_0}u(0^+)+\int_0^{t_1}[t_1-q_0 s]^2 f(s,u(s))d_{q_0}s.
\end{aligned}
$$

Similarly, for $t\in J_1$, applying the operator ${}_{t_1^+}I_{q_1}$ on both sides of $D^3_{q_k}u(t)=f(t,u(t))$, then

$$
D^2_{q_1}u(t)=D^2_{q_1}u(t_1^+)+\int_{t_1}^t f(s,u(s))d_{q_1}s.
$$

In view of impulses conditions in (1) , it holds

$$D_{q_1}^2 u(t) = D_{q_0}^2 u(0^+) + L_1(u(t_1)) + \int_0^{t_1} f(s,u(s)) d_{q_0} s + \int_{t_1}^{t} f(s,u(s)) d_{q_1} s$$

$$D_{q_1} u(t) = D_{q_0} u(0^+) + t D_{q_0}^2 u(0^+) + J_1(u(t_1)) + (t-t_1) L_1(u(t_1)) + (t-t_1)\int_0^{t_1} f(s,u(s)) d_{q_0} s$$
$$+ \int_0^{t_1} [t_1 - q_0 s] f(s,u(s)) d_{q_0} s + \int_{t_1}^{t} [(t-t_1) - q_1(s-t_1)] f(s,u(s)) d_{q_1} s$$

$$u(t) = u(0^+) + t D_{q_0} u(0^+) + (t^2 - t t_1 + t_1^2) D_{q_0}^2 u(0^+) + I_1(u(t_1)) + (t-t_1) J_1(u(t_1)) + (t-t_1)^2 L_1(u(t_1))$$
$$+ \int_0^{t_1} [t_1 - q_0 s]^2 f(s,u(s)) d_{q_0} s + (t-t_1)\int_0^{t_1} [t_1 - q_0 s] f(s,u(s)) d_{q_0} s + (t-t_1)^2 \int_0^{t_1} f(s,u(s)) d_{q_0} s$$
$$+ \int_{t_1}^{t} [(t-t_1) - q_1(s-t_1)] f(s,u(s)) d_{q_1} s.$$

Repeating the above process and using the boundary value conditions given in (1), we can get (2).

Conversely, assume that u satisfies the impulsive $q_k -$ integral equation (1), applying D_{q_k} on both sides of (2) and substituting $t=0$ in (2), then (1) holds. This completes the proof.

MAIN RESULTS

We introduce an operator $Q: PC^2(J,\mathbb{R}) \to PC^2(J,\mathbb{R})$ as $(Qu)(t) = \Gamma[u(t), f(t,u(t))]$. then

$$(D_{q_k} Qu)(t) = -\frac{1}{2}\left[\sum_{i=1}^{p} J_i(u(t_i)) + \sum_{i=1}^{p} (t-t_i) L_i(u(t_i)) + \sum_{i=0}^{p-1} (t-t_{i+1}) \int_{t_i}^{t_{i+1}} f(s,u(s)) dq_i s \right.$$
$$\left. + \sum_{i=0}^{p-1} \int_{t_i}^{t_{i+1}} [(t_{i+1}-t_i) - q_i(s-t_i)] f(s,u(s)) dq_i s + \int_{t_p}^{1} [(1-t_p) - q_p(s-t_p)] f(s,u(s)) dq_p s \right]$$
$$+ \sum_{i=1}^{k} J_i(u(t_i)) + \sum_{i=1}^{k} (t-t_i) L_i(u(t_i)) + \sum_{i=0}^{k-1} (t-t_{i+1}) \int_{t_i}^{t_{i+1}} f(s,u(s)) dq_i s$$
$$+ \sum_{i=0}^{k-1} \int_{t_i}^{t_{i+1}} [(t_{i+1}-t_i) - q_i(s-t_i)] f(s,u(s)) dq_i s + \int_{t_k}^{t} [(t-t_k) - q_k(s-t_k)] f(s,u(s)) dq_k s \quad . \tag{3}$$

And

$$(D_{q_k}^2 Qu)(t) = -\frac{1}{2}\left[\sum_{i=1}^{p} L_i(u(t_i)) + \sum_{i=0}^{p-1} \int_{t_i}^{t_{i+1}} f(s,u(s)) dq_i s + \int_{t_p}^{1} f(s,u(s)) dq_p s \right]$$
$$+ \sum_{i=1}^{k} L_i(u(t_i)) + \sum_{i=0}^{k-1} \int_{t_i}^{t_{i+1}} f(s,u(s)) dq_i s + \int_{t_k}^{t} f(s,u(s)) dq_k s. \tag{4}$$

Then, the equation (1) has a solution if and only if the operator equation Qu has a fixed point.

Theorem 2. Assume the following.

(H_1) There exist nonnegative bounded function $M_i(t)$ $(i=1,2)$ $(\sup_{t\in J}|M_i(t)| = M_i)$ such that

$$|f(t,u)| \le M_1(t) + M_2(t)|u|, \quad \text{for any } t \in J, u \in \mathbb{R}, \; i=1,2.$$

(H_2) For any $u \in \mathbb{R}$, there exist positive constants $\bar{L}, \hat{L}, L'$ such that

$$|I_k(u)| \le \bar{L}, \quad |J_k(u)| < \tilde{L}, \quad |L_k(u)| \le L', \qquad k = 1, 2, \ldots, p \tag{5}$$

Then the problem (1) has at least one solution provided

$$\tau = \sup_{t \in J}\left[M_2(t)\left(\sum_{i=0}^{p-1}\tau_{1i} + \sum_{i=0}^{p-2}\tau_{2i}\tau_{1i} + \sum_{i=0}^{p-3}\sum_{j=i+2}^{p-1}\tau_{3i}\tau_{1i} + (1-q_i)\sum_{i=0}^{p}\tau_{4i} + \sum_{i=0}^{p}\tau_{5i} + \sum_{i=0}^{p-1}\tau_{6i}\right)\right] < 1 .$$

Where $\tau_{1i} = (t_{i+1} - t_i)$, $\tau_{2i} = (t_p - t_{i+1})$, $\tau_{3i} = (t_{j+1} - t_j)(t_j - t_{i+1})$, $\tau_{4i} = (t_{i+1} - t_i)^3 / 1 + q_i$,

$\tau_{5i} = q_i^2 (t_{i+1} - t_i)^3 / 1 + q_i + q_i^2$, $\tau_{6i} = (t_{i+1} - t_i)^2 / 1 + q_i$.

Proof. Firstly, we prove the operator $Q: PC^2(J, \mathbb{R}) \to PC^2(J, \mathbb{R})$ is completely continuous. Clearly, continuity of the operator Q follows from the continuity of f, I_k, J_k and L_k. Let $\Omega \in PC^2(J, \mathbb{R})$ be bounded. Then $\forall t \in J$, $u \in \Omega$, there exist positive constants L_i $(i = 1, 2, 3, 4)$ such that $|f(t,u)| \le L_1$, $|I_k(u)| \le L_2$, $|J_k(u)| \le L_3$, $|L_k(u)| \le L_4$. Thus, we may easily obtain is $\| Qu \|$ bounded. Furthermore, for any $t', t'' \in J_k (k = 0, 1, 2, \ldots, p)$ satisfying $t' < t''$, we have

$$\begin{aligned}
&|(Qu)(t'') - (Qu)(t')| \\
&\le \frac{1}{2}\Bigg[+ \sum_{i=1}^{p}(t'' - t_i)|J_i(u(t_i))| + \sum_{i=1}^{p}(t'' - t_i)^2 |L_i(u(t_i))| + \sum_{i=1}^{p-1}(t'' - t_p)(t_p - t_i)|L_i(u(t_i))| + \sum_{i=0}^{p-1}(t'' - t_{i+1})^2 \int_{t_i}^{t_{i+1}} |f(s,u(s))| d_{q_i}s \\
&+ \sum_{i=0}^{p-2}(t'' - t_p)\tau_{2i}\int_{t_i}^{t_{i+1}} |f(s,u(s))| d_{q_i}s + \sum_{i=0}^{p-1}(t'' - t_{i+1})\int_{t_i}^{t_{i+1}} [\tau_{1i} - q_i(s - t_i)] |f(s,u(s))| d_{q_i}s \Bigg] + \sum_{i=1}^{k}(t'' - t_i)|J_i(u(t_i))| \\
&+ \sum_{i=1}^{k}(t'' - t_i)^2 |L_i(u(t_i))| + \sum_{i=1}^{k-1}(t'' - t_k)(t_k - t_i)|L_i(u(t_i))| + \sum_{i=0}^{k-1}(t'' - t_{i+1})^2 \int_{t_i}^{t_{i+1}} |f(s,u(s))| d_{q_i}s \\
&+ \sum_{i=0}^{k-2}(t'' - t_k)(t_k - t_{i+1})\int_{t_i}^{t_{i+1}} |f(s,u(s))| d_{q_i}s + \sum_{i=0}^{k-1}(t'' - t_{i+1})\int_{t_i}^{t_{i+1}} [\tau_{1i} - q_i(s - t_i)] |f(s,u(s))| d_{q_i}s \\
&+ \int_{t_k}^{t''} [(t'' - t_k) - q_k(s - t_k)]^2 |f(s,u(s))| d_{q_k}s - \frac{1}{2}\Bigg[+ \sum_{i=1}^{p}(t' - t_i)|J_i(u(t_i))| + \sum_{i=1}^{p}(t' - t_i)^2 |L_i(u(t_i))| \\
&+ \sum_{i=1}^{p-1}(t' - t_p)(t_p - t_i)|L_i(u(t_i))| + \sum_{i=0}^{p-1}(t' - t_{i+1})^2 \int_{t_i}^{t_{i+1}} |f(s,u(s))| d_{q_i}s + \sum_{i=0}^{p-2}(t' - t_p)\tau_{2i}\int_{t_i}^{t_{i+1}} |f(s,u(s))| d_{q_i}s \\
&+ \sum_{i=0}^{p-1}(t' - t_{i+1})\int_{t_i}^{t_{i+1}} [\tau_{1i} - q_i(s - t_i)] |f(s,u(s))| d_{q_i}s \Bigg] - \sum_{i=1}^{k}(t' - t_i)|J_i(u(t_i))| - \sum_{i=1}^{k}(t' - t_i)^2 |L_i(u(t_i))| \\
&- \sum_{i=1}^{k-1}(t' - t_k)(t_k - t_i)|L_i(u(t_i))| - \sum_{i=0}^{k-1}(t' - t_{i+1})^2 \int_{t_i}^{t_{i+1}} |f(s,u(s))| d_{q_i}s - \sum_{i=0}^{k-2}(t' - t_k)(t_k - t_{i+1})\int_{t_i}^{t_{i+1}} |f(s,u(s))| d_{q_i}s \\
&- \sum_{i=0}^{k-1}(t' - t_{i+1})\int_{t_i}^{t_{i+1}} [\tau_{1i} - q_i(s - t_i)] |f(s,u(s))| d_{q_i}s - \int_{t_k}^{t'} [(t' - t_k) - q_k(s - t_k)]^2 |f(s,u(s))| d_{q_k}s
\end{aligned}$$

$$\leq \frac{1}{2}\left[\sum_{i=1}^{p}(t''-t')|J_i(u(t_i))| + \sum_{i=1}^{p}\left[(t''^2-t'^2)-2t_k(t''-t')\right]|L_i(u(t_i))| + \sum_{i=1}^{p-1}(t''-t')(t_p-t_i)|L_i(u(t_i))|\right]$$
$$+\sum_{i=1}^{k}(t''-t')|J_i(u(t_i))| + \sum_{i=1}^{k}\left[(t''^2-t'^2)-2t_k(t''-t')\right]|L_i(u(t_i))| + \sum_{i=1}^{k-1}(t''-t')(t_k-t_i)|L_i(u(t_i))|$$
$$+\frac{1}{2}\left[\sum_{i=0}^{p-1}\left[(t''^2-t'^2)-2t_{i+1}(t''-t')\right]\int_{t_i}^{t_{i+1}}|f(s,u(s))|\,d_{q_i}s + \sum_{i=0}^{p-2}(t''-t')\tau_{2i}\int_{t_i}^{t_{i+1}}|f(s,u(s))|\,d_{q_i}s\right.$$
$$\left.+\sum_{i=0}^{p-1}(t''-t')\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]|f(s,u(s))|\,d_{q_i}s\right] + \sum_{i=0}^{k-1}\left[(t''^2-t'^2)-2t_{i+1}(t''-t')\right]\int_{t_i}^{t_{i+1}}|f(s,u(s))|\,d_{q_i}s$$
$$+\sum_{i=0}^{k-2}(t''-t')(t_k-t_{i+1})\int_{t_i}^{t_{i+1}}|f(s,u(s))|\,d_{q_i}s + \sum_{i=0}^{k-1}(t''-t')\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]|f(s,u(s))|\,d_{q_i}s$$
$$+\int_{t'}^{t''}[(t''-t_k)-q_k(s-t_k)]^2|f(s,u(s))|\,d_{q_k}s + \int_{t_k}^{t'}\left[(t''^2-t'^2)-2t_k(t''-t')-2q_k(s-t_k)(t''-t')\right]|f(s,u(s))|\,d_{q_k}s$$

$$\leq \frac{1}{2}\left[p(t''-t')|J_i(u(t_i))| + p\left[(t''^2-t'^2)-2t_k(t''-t')\right]|L_i(u(t_i))| + p(t''-t')(t_p-t_i)|L_i(u(t_i))|\right]$$
$$+p(t''-t')|J_i(u(t_i))| + p\left[(t''^2-t'^2)-2t_k(t''-t')\right]|L_i(u(t_i))| + p(t''-t')(t_p-t_i)|L_i(u(t_i))|$$
$$+\frac{1}{2}\left[\sum_{i=0}^{p-1}\left[(t''^2-t'^2)-2t_{i+1}(t''-t')\right]\int_{t_i}^{t_{i+1}}|f(s,u(s))|\,d_{q_i}s + \sum_{i=0}^{p-2}(t''-t')\tau_{2i}\int_{t_i}^{t_{i+1}}|f(s,u(s))|\,d_{q_i}s\right.$$
$$\left.+\sum_{i=0}^{p-1}(t''-t')\int_{t_i}^{t_{i+1}}\tau_{1i}|f(s,u(s))|\,d_{q_i}s\right] + \sum_{i=0}^{k-1}\left[(t''^2-t'^2)-2t_{i+1}(t''-t')\right]\int_{t_i}^{t_{i+1}}|f(s,u(s))|\,d_{q_i}s$$
$$+\sum_{i=0}^{k-2}(t''-t')(t_k-t_{i+1})\int_{t_i}^{t_{i+1}}|f(s,u(s))|\,d_{q_i}s + \sum_{i=0}^{k-1}(t''-t')\int_{t_i}^{t_{i+1}}\tau_{1i}|f(s,u(s))|\,d_{q_i}s$$
$$+\int_{t'}^{t''}(t''-t_k)^2|f(s,u(s))|\,d_{q_k}s + \int_{t_k}^{t'}\left[(t''^2-t'^2)-2t_k(t''-t')\right]|f(s,u(s))|\,d_{q_k}s$$
$$\leq \frac{3}{2}\left[pL_3-(t_i+t_k)pL_4+\sum_{i=0}^{p-1}2t_{i+1}\tau_{1i}L_1+\sum_{i=0}^{p-2}(t_k-t_{i+1})\tau_{1i}L_1+\sum_{i=0}^{p-1}\tau_{1i}^2L_1+(t''-t_k)^2L_1-2t_k(t'-t_k)L_1\right](t''-t')$$
$$+\frac{3}{2}\left[pL_4+\sum_{i=0}^{p-1}\tau_{1i}L_1+(t'-t_k)L_1\right](t''^2-t'^2)$$

Similarly, we can get

$$\left|(D_{q_k}Qu)(t'')-(D_{q_k}Qu)(t')\right| \leq \left[\frac{3}{2}pL_4+\frac{3}{2}\sum_{i=0}^{p-1}\tau_{1i}L_1+(t''-t_k)L_1+(t'-t_k)L_1\right](t''-t').$$

$$\left|(D_{q_k}^2Qu)(t'')-(D_{q_k}^2Qu)(t')\right| \leq (t''-t')L_1\,.$$

As $t'\to t''$, the right hand side of the above inequality tends to zero. Thus, $Q(\Omega)$ is relatively compact. As a consequence of Arzela Ascoli's theorem, Q is a compact operator. Therefore, Q is a completely continuous operator. Let

$$W=\{u\in PC^2(J,\mathbb{R})|u=\lambda Qu, 0<\lambda<1\}.$$

Next, we show W is bounded. Let $u\in W$ then $u=\lambda Qu$, $0<\lambda<1$. Denote $M_1+M_2\|u\|=\eta$. For any $t\in J$ by conditions (H_1) and (H_2), we have

$$
\begin{aligned}
&|u(t)|=\lambda|(Qu)(t)|\\
&\le\frac{1}{2}\Bigg[\sum_{i=1}^{p}|I_i(u(t_i))|+\sum_{i=1}^{p}(t-t_i)|J_i(u(t_i))|+\sum_{i=1}^{p}(t-t_i)^2|L_i(u(t_i))|+\sum_{i=1}^{p-1}(t-t_P)(t_p-t_i)|L_i(u(t_i))|\\
&+\sum_{i=1}^{p-2}\sum_{j=i+1}^{p-1}(t_{j+1}-t_j)(t_j-t_i)|L_i(u(t_i))|+\sum_{i=0}^{p-1}(t-t_{i+1})^2\int_{t_i}^{t_{i+1}}|f(s,u(s))|d_{q_i}s+\sum_{i=0}^{p-2}(t-t_P)\tau_{2i}\int_{t_i}^{t_{i+1}}|f(s,u(s))|d_{q_i}s\\
&+\sum_{i=0}^{p-3}\sum_{j=i+2}^{p-1}\tau_{3i}\int_{t_i}^{t_{i+1}}|f(s,u(s))|d_{q_i}s+\sum_{i=0}^{p-1}(t-t_{i+1})\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]|f(s,u(s))|d_{q_i}s\\
&+\sum_{i=0}^{p-1}\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]^2|f(s,u(s))|d_{q_i}s+\int_{t_p}^{1}[(1-t_p)-q_p(s-t_p)]^2|f(s,u(s))|d_{q_p}s\Bigg]\\
&+\sum_{i=1}^{k}|I_i(u(t_i))|+\sum_{i=1}^{k}(t-t_i)|J_i(u(t_i))|+\sum_{i=1}^{k}(t-t_i)^2|L_i(u(t_i))|+\sum_{i=1}^{k-1}(t-t_k)(t_k-t_i)|L_i(u(t_i))|\\
&+\sum_{i=1}^{k-2}\sum_{j=i+1}^{k-1}(t_{j+1}-t_j)(t_j-t_i)|L_i(u(t_i))|+\sum_{i=0}^{k-1}(t-t_{i+1})^2\int_{t_i}^{t_{i+1}}|f(s,u(s))|d_{q_i}s+\sum_{i=0}^{k-2}(t-t_k)(t_k-t_{i+1})\int_{t_i}^{t_{i+1}}|f(s,u(s))|d_{q_i}s\\
&+\sum_{i=0}^{k-3}\sum_{j=i+2}^{k-1}\tau_{3i}\int_{t_i}^{t_{i+1}}|f(s,u(s))|d_{q_i}s+\sum_{i=0}^{k-1}(t-t_{i+1})\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]|f(s,u(s))|d_{q_i}s\\
&+\sum_{i=0}^{k-1}\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]^2|f(s,u(s))|d_{q_i}s+\int_{t_k}^{t}[(t-t_k)-q_k(s-t_k)]^2|f(s,u(s))|d_{q_k}s
\end{aligned}
$$

$$
\begin{aligned}
&\le\frac{1}{2}\Bigg[\sum_{i=1}^{p}\bar{L}+\sum_{i=1}^{p}(t-t_i)\tilde{L}+\sum_{i=1}^{p}(t-t_i)^2L'+\sum_{i=1}^{p-1}(t-t_P)(t_p-t_i)L'+\sum_{i=1}^{p-2}\sum_{j=i+1}^{p-1}(t_{j+1}-t_j)(t_j-t_i)L'\\
&+\sum_{i=0}^{p-1}(t-t_{i+1})^2\int_{t_i}^{t_{i+1}}\eta d_{q_i}s+\sum_{i=0}^{p-2}(t-t_P)\tau_{2i}\int_{t_i}^{t_{i+1}}\eta d_{q_i}s+\sum_{i=0}^{p-3}\sum_{j=i+2}^{p-1}\tau_{3i}\int_{t_i}^{t_{i+1}}\eta d_{q_i}s+\sum_{i=0}^{p-1}(t-t_{i+1})\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]\eta d_{q_i}s\\
&+\sum_{i=0}^{p-1}\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]^2\eta d_{q_i}s+\int_{t_p}^{1}[(1-t_p)-q_p(s-t_p)]^2\eta d_{q_p}s\Bigg]+\sum_{i=1}^{k}\bar{L}+\sum_{i=1}^{k}(t-t_i)\tilde{L}+\sum_{i=1}^{k}(t-t_i)^2L'\\
&+\sum_{i=1}^{k-1}(t-t_k)(t_k-t_i)L'+\sum_{i=1}^{k-2}\sum_{j=i+1}^{k-1}(t_{j+1}-t_j)(t_j-t_i)L'+\sum_{i=0}^{k-1}(t-t_{i+1})^2\int_{t_i}^{t_{i+1}}\eta d_{q_i}s+\sum_{i=0}^{k-2}(t-t_k)(t_k-t_{i+1})\int_{t_i}^{t_{i+1}}\eta d_{q_i}s+\sum_{i=0}^{k-3}\sum_{j=i+2}^{k-1}\tau_{3i}\int_{t_i}^{t_{i+1}}\eta d_{q_i}s\\
&+\sum_{i=0}^{k-1}(t-t_{i+1})\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]\eta d_{q_i}s+\sum_{i=0}^{k-1}\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]^2\eta d_{q_i}s+\int_{t_k}^{t}[(t-t_k)-q_k(s-t_k)]^2\eta d_{q_k}s
\end{aligned}
$$

$$
\begin{aligned}
&\le\frac{1}{2}\Bigg[p\bar{L}+p(t-t_i)\tilde{L}+p(t-t_i)^2L'+p(t-t_P)L'+p^2L'+\sum_{i=0}^{p-1}(t-t_{i+1})^2\tau_{1i}(M_1+M_2\|u\|)\\
&+\sum_{i=0}^{p-2}(t-t_P)\tau_{2i}\tau_{1i}(M_1+M_2\|u\|)+\sum_{i=0}^{p-3}\sum_{j=i+2}^{p-1}\tau_{3i}\tau_{1i}(M_1+M_2\|u\|)+\sum_{i=0}^{p-1}(t-t_{i+1})(\tau_{1i}^2-q_i\tau_{6i})(M_1+M_2\|u\|)\\
&+\sum_{i=0}^{p-1}(\tau_{1i}^3+\tau_{5i}-2q_i\tau_{4i})(M_1+M_2\|u\|)+(1-t_p)^3(M_1+M_2\|u\|)+q_p^2(1-t_p)^3(M_1+M_2\|u\|)/(1+q_p+q_p^2)\\
&-2q_p(1-t_p)^3(M_1+M_2\|u\|)/1+q_p\Bigg]+p\bar{L}+p(t-t_i)\tilde{L}+p(t-t_i)^2L'+p(t-t_k)L'+p^2L'\\
&+\sum_{i=0}^{k-1}(t-t_{i+1})^2\tau_{1i}(M_1+M_2\|u\|)+\sum_{i=0}^{k-2}(t-t_k)(t_k-t_{i+1})\tau_{1i}(M_1+M_2\|u\|)+\sum_{i=0}^{k-3}\sum_{j=i+2}^{k-1}\tau_{3i}\tau_{1i}(M_1+M_2\|u\|)\\
&+\sum_{i=0}^{k-1}(t-t_{i+1})(\tau_{1i}^2-q_i\tau_{6i})(M_1+M_2\|u\|)+\sum_{i=0}^{k-1}(\tau_{1i}^3+\tau_{5i}-2q_i\tau_{4i})(M_1+M_2\|u\|)+(t-t_k)^3(M_1+M_2\|u\|)\\
&+q_k^2(t-t_k)^3(M_1+M_2\|u\|)/(1+q_k+q_k^2)-2q_k(t-t_k)^3(M_1+M_2\|u\|)/1+q_k
\end{aligned}
$$

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$$+\sum_{i=0}^{k-1}(t-t_{i+1})(\tau_{1i}^2-q_i\tau_{6i})(M_1+M_2\|u\|)+\sum_{i=0}^{k-1}(\tau_{1i}^3+\tau_{5i}-2q_i\tau_{4i})(M_1+M_2\|u\|)+(t-t_k)^3(M_1+M_2\|u\|)$$

$$+q_k^2(t-t_k)^3(M_1+M_2\|u\|)/(1+q_k+q_k^2)-2q_k(t-t_k)^3(M_1+M_2\|u\|)/1+q_k$$

$$\le\frac{3}{2}\left(p\bar{L}+p\tilde{L}+2pL'+p^2L'\right)+\frac{3}{2}M_1\left(\sum_{i=0}^{p-1}\tau_{1i}+\sum_{i=0}^{p-2}\tau_{2i}\tau_{1i}+\sum_{i=0}^{p-3}\sum_{j=i+2}^{p-1}\tau_{3i}\tau_{1i}+\sum_{i=0}^{p}(1-q_i)\tau_{4i}+\sum_{i=0}^{p}\tau_{5i}+\sum_{i=0}^{p-1}\tau_{6i}\right)$$

$$+\left[\frac{1}{2}M_2\left(\sum_{i=0}^{p-1}\tau_{1i}+\sum_{i=0}^{p-2}\tau_{2i}\tau_{1i}+\sum_{i=0}^{p-3}\sum_{j=i+2}^{p-1}\tau_{3i}\tau_{1i}+\sum_{i=0}^{p}(1-q_i)\tau_{4i}+\sum_{i=0}^{p}\tau_{5i}+\sum_{i=0}^{p-1}\tau_{6i}\right)\right]\|u\|.$$

$$\|u\|\le\frac{1}{1-\tau}(M_1\left(\sum_{i=0}^{p-1}\tau_{1i}+\sum_{i=0}^{p-2}\tau_{2i}\tau_{1i}+\sum_{i=0}^{p-3}\sum_{j=i+2}^{p-1}\tau_{3i}\tau_{1i}+\sum_{i=0}^{p}(1-q_i)\tau_{4i}+\sum_{i=0}^{p}\tau_{5i}+\sum_{i=0}^{p-1}\tau_{6i}\right)+p\bar{L}+p\tilde{L}+2pL'+p^2L').$$

Similarly, For any $t\in J$ by conditions (H_1) and (H_2), we may show that $\|\mathrm{D}_{q_k}u\|,\|\mathrm{D}_{q_k}^2u\|$ area bounded. So, the set W is bounded. Thus, Schauder fixed point theorem ensures the impulsive q_k-integrodifference equation (1) has at least one solution.

Corollary 4. Assume that there exist nonnegative constants L_i $(i=1,2,3,4)$ such that

$$|f(t,u)|\le L_1,\quad I_k(u(t_k))\le L_2,\quad J_k(u(t_k))\le L_3,\quad |L_k(u)|\le L_4,$$

for any $t\in J$, $u\in\mathbb{R}$, $k=1,2,\dots p$. Then problem (1) has at least one solution.

Theorem 5. Assume the following.

(H_4) There exist nonnegative bounded functions $M(t)$ such that

$$|f(t,u)-f(t,v)|\le M(t)|u-v|,\qquad \text{for } t\in J,\ u,v\in\mathbb{R}.$$

(H_5) Let $u,v\in\mathbb{R}$ There exist positive constants K, G, X such that

$$|I_k(u)-I_k(v)|\le K|u-v|,|J_k(u)-J_k(v)|\le G|u-v|,|L_k(u)-L_k(v)|\le X|u-v|,\ k=1,2,\dots,p.$$

$$(H_6)\ K=\sup_{t\in J}\left\{\frac{3}{2}\left[pK+pG+2pX+p^2X+M\left(\sum_{i=0}^{p-1}\tau_{1i}-\sum_{i=0}^{p-2}\tau_{2i}\tau_{1i}-\sum_{i=0}^{p-3}\sum_{j=i+2}^{p-1}\tau_{3i}\tau_{1i}+\sum_{i=0}^{p}(1-q_i)\tau_{4i}+\sum_{i=0}^{p}\tau_{5i}+\sum_{i=0}^{p-1}\tau_{6i}\right)\right]\right\}<1.$$

Then problem (1) has a unique solution.

Proof. Clearly Q is a continuous operator. Denote $\sup_{t\in J}|M(t)|=M$. For $\forall u,v\in PC^2(J,\mathbb{R})$, by (H_4) and (H_5), we have

$$|(Qu)(t)-(Qv)(t)|$$

$$\leq \frac{1}{2}\Bigg[\sum_{i=1}^{p} |I_i(u)-I_i(v)| + \sum_{i=1}^{p}(t-t_i)|J_i(u)-J_i(v)| + \sum_{i=1}^{p}(t-t_i)^2|L_i(u)-L_i(v)| + \sum_{i=1}^{p-1}(t-t_P)(t_p-t_i)|L_i(u)-L_i(v)|$$

$$+\sum_{i=1}^{p-2}\sum_{j=i+1}^{p-1}(t_{j+1}-t_j)(t_j-t_i)|L_i(u)-L_i(v)| + \sum_{i=0}^{p-1}(t-t_{i+1})^2\int_{t_i}^{t_{i+1}}|f(s,u(s))-f(s,v(s))|d{q_i}s$$

$$+\sum_{i=0}^{p-2}(t-t_P)\tau_{2i}\int_{t_i}^{t_{i+1}}|f(s,u(s))-f(s,v(s))|d{q_i}s + \sum_{i=0}^{p-3}\sum_{j=i+2}^{p-1}\tau_{3i}\int_{t_i}^{t_{i+1}}|f(s,u(s))-f(s,v(s))|d{q_i}s$$

$$+\sum_{i=0}^{p-1}(t-t_{i+1})\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]|f(s,u(s))-f(s,v(s))|d{q_i}s + \sum_{i=0}^{p-1}\int_{t_i}^{t_{i+1}}[(\tau_{1i}-t_i)-q_i(s-t_i)]^2|f(s,u(s))-f(s,v(s))|d{q_i}s$$

$$+\int_{t_p}^{1}[(1-t_p)-q_p(s-t_p)]^2|f(s,u(s))-f(s,v(s))|d{q_p}s\Bigg] + \sum_{i=1}^{k}|I_i(u)-I_i(v)| + \sum_{i=1}^{k}(t-t_i)|J_i(u)-J_i(v)|$$

$$+\sum_{i=1}^{k}(t-t_i)^2|L_i(u)-L_i(v)| + \sum_{i=1}^{k-1}(t-t_k)(t_k-t_i)|L_i(u)-L_i(v)| + \sum_{i=1}^{k-2}\sum_{j=i+1}^{k-1}(t_{j+1}-t_j)(t_j-t_i)|L_i(u)-L_i(v)|$$

$$+\sum_{i=0}^{k-1}(t-t_{i+1})^2\int_{t_i}^{t_{i+1}}|f(s,u(s))-f(s,v(s))|d{q_i}s + \sum_{i=0}^{k-2}(t-t_k)(t_k-t_{i+1})\int_{t_i}^{t_{i+1}}|f(s,u(s))-f(s,v(s))|d{q_i}s$$

$$+\sum_{i=0}^{k-3}\sum_{j=i+2}^{k-1}\tau_{3i}\int_{t_i}^{t_{i+1}}|f(s,u(s))-f(s,v(s))|d{q_i}s + \sum_{i=0}^{k-1}(t-t_{i+1})\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]|f(s,u(s))-f(s,v(s))|d{q_i}s$$

$$+\sum_{i=0}^{k-1}\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]^2|f(s,u(s))-f(s,v(s))|d{q_i}s + \int_{t_k}^{t}[(t-t_k)-q_k(s-t_k)]^2|f(s,u(s))-f(s,v(s))|d{q_k}s$$

$$\leq \frac{1}{2}\Bigg[\sum_{i=1}^{p}K|u-v| + \sum_{i=1}^{p}(t-t_i)G|u-v| + \sum_{i=1}^{p}(t-t_i)^2X|u-v| + \sum_{i=1}^{p-1}(t-t_P)(t_p-t_i)X|u-v|$$

$$+\sum_{i=1}^{p-2}\sum_{j=i+1}^{p-1}(t_{j+1}-t_j)(t_j-t_i)X|u-v| + \sum_{i=0}^{p-1}(t-t_{i+1})^2\int_{t_i}^{t_{i+1}}M(t)d{q_i}s + \sum_{i=0}^{p-2}(t-t_P)\tau_{2i}\int_{t_i}^{t_{i+1}}M(t)d{q_i}s$$

$$+\sum_{i=0}^{p-3}\sum_{j=i+2}^{p-1}\tau_{3i}\int_{t_i}^{t_{i+1}}M(t)d{q_i}s + \sum_{i=0}^{p-1}(t-t_{i+1})\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]M(t)d{q_i}s + \sum_{i=0}^{p-1}\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]^2M(t)d{q_i}s$$

$$+\int_{t_p}^{1}[(1-t_p)-q_p(s-t_p)]^2M(t)d{q_p}s\Bigg] + \sum_{i=1}^{k}K|u-v| + \sum_{i=1}^{k}(t-t_i)G|u-v| + \sum_{i=1}^{k}(t-t_i)^2X|u-v|$$

$$+\sum_{i=1}^{k-1}(t-t_k)(t_k-t_i)X|u-v| + \sum_{i=1}^{k-2}\sum_{j=i+1}^{k-1}(t_{j+1}-t_j)(t_j-t_i)X|u-v| + \sum_{i=0}^{k-1}(t-t_{i+1})^2\int_{t_i}^{t_{i+1}}M(t)d{q_i}s$$

$$+\sum_{i=0}^{k-2}(t-t_k)(t_k-t_{i+1})\int_{t_i}^{t_{i+1}}M(t)d{q_i}s + \sum_{i=0}^{k-3}\sum_{j=i+2}^{k-1}\tau_{3i}\int_{t_i}^{t_{i+1}}M(t)d{q_i}s + \sum_{i=0}^{k-1}(t-t_{i+1})\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]M(t)d{q_i}s$$

$$+\sum_{i=0}^{k-1}\int_{t_i}^{t_{i+1}}[\tau_{1i}-q_i(s-t_i)]^2M(t)d{q_i}s + \int_{t_k}^{t}[(t-t_k)-q_k(s-t_k)]^2M(t)d{q_k}s$$

$$\leq \Bigg\{\frac{1}{2}\Big[pK + p(t-t_i)G + p(t-t_i)^2X + p(t-t_P)X + p^2X + \sum_{i=0}^{p-1}(t-t_{i+1})^2\tau_{1i}M + \sum_{i=0}^{p-2}(t-t_P)\tau_{2i}\tau_{1i}M$$

$$+\sum_{i=0}^{p-3}\sum_{j=i+2}^{p-1}\tau_{3i}\tau_{1i}M + \sum_{i=0}^{p-1}(t-t_{i+1})(\tau_{1i}^2-q_i\tau_{6i})M + \sum_{i=0}^{p-1}(\tau_{1i}^3+\tau_{5i}-2q_i\tau_{4i})M + (1-t_p)^3M$$

$$+q_p^2(1-t_p)^3M/(1+q_p+q_p^2) - 2q_p(1-t_p)^3M/1+q_p\Big] + pK + p(t-t_i)G + p(t-t_i)^2X + p(t-t_k)X$$

$$+p^2X + \sum_{i=0}^{k-1}(t-t_{i+1})^2\tau_{1i}M + \sum_{i=0}^{k-2}(t-t_k)(t_k-t_{i+1})\tau_{1i}M + \sum_{i=0}^{k-3}\sum_{j=i+2}^{k-1}\tau_{3i}\tau_{1i}M + \sum_{i=0}^{k-1}(t-t_{i+1})(\tau_{1i}^2-q_i\tau_{6i})M$$

$$+\sum_{i=0}^{k-1}(\tau_{1i}^3+\tau_{5i}-2q_i\tau_{4i})M+(t-t_k)^3M+q_k^2(t-t_k)^3M/(1+q_k+q_k^2)-2q_k(t-t_k)^3M/1+q_k\Big\}\|u-v\|$$

$$\leq\frac{3}{2}\left[pK+pG+2pX+p^2X+M\left(\sum_{i=0}^{p-1}\tau_{1i}-\sum_{i=0}^{p-2}\tau_{2i}\tau_{1i}-\sum_{i=0}^{p-3}\sum_{j=i+2}^{p-1}\tau_{3i}\tau_{1i}+\sum_{i=0}^{p}(1-q_i)\tau_{4i}+\sum_{i=0}^{p}\tau_{5i}+\sum_{i=0}^{p-1}\tau_{6i}\right)\right]\|u-v\|$$

$$\leq K\,\|u-v\|.$$

As $K<1$ by (H_6). Therefore, Q is a contractive map. Thus, the Theorem 5 holds.

EXAMPLE

Consider the following third order nonlinear q_k-integrodifference equation with impulses

$$D_{1/(2+k)}^3=8+3\sqrt{t}+\ln(1+5t^3+\frac{t^2}{5}|u(t)|),\qquad t\in(0,1),\ t\neq\frac{1}{1+2k},\tag{6}$$

$$D_{1/(2+k)}^2u\left(\frac{1}{1+2k}\right)-D_{1/(1+k)}^2u\left(\frac{1}{1+2k}\right)=-\cos\left(u\left(\frac{1}{1+2k}\right)\right),\tag{7}$$

$$D_{1/(2+k)}u\left(\frac{1}{1+2k}\right)-D_{1/(1+k)}u\left(\frac{1}{1+2k}\right)=\sin\left(u\left(\frac{1}{1+2k}\right)\right),\tag{8}$$

$$\Delta u\left(\frac{1}{1+2k}\right)=\cos\left(u\left(\frac{1}{1+2k}\right)\right),\qquad k=1,2,\cdots,6\tag{9}$$

$$u(0)=-u(1),\quad D_{1/2}u(0)=-D_{1/(3+P)}u(1),\quad D_{1/2}^2u(0)=-D_{1/(3+P)}^2u(1).\tag{10}$$

Obviously, $q_k=1/(2+k)(k=0,1,2,\cdots,6)$, $t_k=1/(1+2k)(k=1,2,\cdots,6)$, $f(t,u)=8+3\sqrt{t}+\ln(1+5t^3+\frac{t^2}{5}|u(t)|)$, $I_k(u)=\cos u$, $J_k(u)=\sin u$ and $L_k(u)=-\cos u$. By a simple calculation, we have

$$|f(t,u)|\leq 8+3\sqrt{t}+5t^3+\frac{t^2}{5}|u(t)|,\quad |I_k(u)|\leq 1,\quad |J_k(u)|\leq 1,\quad |L_k(u)|\leq 1.$$

Take $M_1(t)=8+3\sqrt{t}+5t^3$, $M_2(t)=\frac{t^2}{5}$, $M_3(t)=10t$, $M_4(t)=\frac{t^3}{3}$, and $\bar{L}=\hat{L}=L'=1$.

Then all conditions of the Theorem 3 satisfy, the above problem (6)-(10) has at least one solution.

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